Towards Bayesian Optical Flow

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 $BEng(Hons \ 1)(New castle)$

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Statement of originality

I hereby certify that the work embodied in the thesis is my own work, conducted under normal supervision. The thesis contains no material which has been accepted, or is being examined, for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I give consent to the final version of my thesis being made available worldwide when deposited in the University's Digital Repository, subject to the provisions of the Copyright Act 1968 and any approved embargo.

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I hereby certify that the work embodied in this thesis contains published paper/s/scholarly work of which I am a joint author. I have included as part of the thesis a written declaration endorsed in writing by my supervisors, attesting to my contribution to the joint publication/s/scholarly work.



Friday March 31st 2023

Acknowledgement of Authorship

By signing below I confirm that Timothy Farnworth contributed all theory to the paper/ publication entitled "A heteroscedastic likelihood model for two-frame optical flow".

Yours sincerely,

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Abstract

Autonomous systems play a crucial role in various aspects of modern transportation, agriculture, and healthcare. Navigation is a critical component of autonomous systems, enabling them to understand their position and interact with the surrounding environment. This requires the use of different sensing technologies such as GPS, IMU, and vision. Among these, vision sensors are exceedingly versatile and rich in information. One of the primary channels of visual information used in nature is observed motion in the scene, known as optical flow. With a brain weighing less than a milligram, bees exploit optical flow to successfully navigate to and from food sources that may be located 10km away from their hive. Considering its tiny brain, researchers regard the bee as the minimal working example of a complete visual navigation solution.

Decades of research have sought to replicate the bee's remarkable navigation capabilities in engineered solutions. In practice, optical flow is measured by applying algorithms to image sequences captured by cameras and has been leveraged in computer science and engineering to improve position and orientation (pose) estimates of navigation systems. However, the variability of optical flow measurements is difficult to characterise, as it depends not only on the imaging sensor but also on environmental lighting and texture, as well as the algorithms employed. Previous models have assumed a Gaussian noise distribution, which is convenient for posing egomotion estimation as a Gaussian least-squares problem. However, recent research has shown that this assumption can be inadequate, leading to errors in pose estimation and uncertainty measures. Consequently, there are ongoing efforts to develop new models and algorithms that improve the estimates of pose and motion in autonomous systems that use vision-based navigation.

This thesis aims to address these challenges. Firstly, a novel texture-based likelihood model for optical flow is developed that utilises a Laplace-Cauchy mixture (LCM) distribution. The likelihood adheres to peaks and tails of the optical flow error distributions observed in empirical data, whereas the traditional Gaussian assumption is restrictive in describing this phenomenon. Compared to the empirical characteristics, the LCM distribution has 94% less error than the traditional Gaussian model and 48% less error than contemporary models, according to the Kolmogorov-Smirnov statistic. It is demonstrated on the KITTI dataset to improve visual odometry estimation accuracy and minimise position drift by at least 39%, surpassing the performance of previous models. Secondly, to use the novel LCM likelihood model within a Gaussian state estimator, we present VIVO, an approximate Bayes visual odometry solution based on variational inference. VIVO fuses information from a Wiener motion prior and optical flow measurements via the LCM likelihood model. This leads to a 43% reduction in drift compared to maximum likelihood approaches with the same likelihood model. Results show that VIVO results in a 57%–71% reduction in estimator position drift compared to contemporary methods.

The presented work ushers the field of robotics toward a Bayesian treatment of optical flow measurements enabling the fusion of flexible non-Gaussian optical flow error models within a Gaussian assumed density filter. The development of the LCM likelihood model reduces the chasm between approximate models and empirical data, enabling justifiable reasoning with optical flow measurements. Overall, the work presented in this thesis enhances the componentry required to perform Bayesian sensor fusion with vision data, enabling the inclusion of vision with sensors such as GNSS, LiDAR and IMU, strengthening trusted autonomy.

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